Double beta transition mechanism

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Abstract

After briefly reviewing $\beta\beta$ decay as a test of the neutrino mass, I examine the nuclear structure involved in this process. Simple formulas (à la Padé) are designed for the transition amplitudes and the general behavior of $\beta\beta$ decay amplitudes in the quasiparticle random phase approximation are discussed. Results of a calculation for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te and ^{130}Te nuclei are presented, in which the particle-particle interaction strengths have been fixed by invoking the partial restoration of the isospin and Wigner SU(4) symmetries. An upper limit of $<m_\nu> \approx 1$ eV is obtained for the effective neutrino mass.

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1. Introduction

The double beta ($\beta\beta$) decay is a nice example of the interrelation between the Particle Physics and the Nuclear Physics: *we can get information on the properties of the neutrino and the weak interaction from the $\beta\beta$ decay only if we know who to deal we the nuclear structure involved in the process.* There are already several well-known reviews on the neutrino physics [1, 2] and therefore, after a brief historical overview, I will limit this talk to the nuclear facet of the problem. More precisely, some recent developments performed by our group will be summarized.

Because of the pairing force there are approximately 50 nuclear systems in which the odd-odd isobar, within the isobaric triplet (A, Z), $(A, Z + 1)$, $(A, Z + 2)$, has a higher mass than its neighbors. Within such a scenario the single β decay, is energetically forbidden and the initial nucleus can disintegrate only via the $\beta\beta$ decay. This is a second-order weak interaction process, similar to electromagnetic processes such as the atomic Raman scattering and the nuclear $\gamma\gamma$ decay [3].

The modes by which $\beta\beta$ decay can take place are connected with the neutrino (ν) - antineutrino ($\tilde{\nu}$) distinction. If ν and $\tilde{\nu}$ are defined by the transitions:

$$n \longrightarrow p + e^- + \tilde{\nu} \tag{1}$$
$$\nu + n \longrightarrow p + e^-,$$

the decay $(A, Z) \longrightarrow (A, Z + 2)$ can occur by successive β decays:

$$(A, Z) \longrightarrow (A, Z + 1) + e^- + \tilde{\nu}$$
$$\longrightarrow (A, Z + 2) + 2e^- + 2\tilde{\nu}, \tag{2}$$

passing through the intermediate virtual states of the $(A, Z + 1)$ nucleus.

Yet the neutrino is the only fermion in lacking a additionally conserved quantum number that differences between ν and $\tilde{\nu}$. Thus it is possible that the neutrino is a Majorana particle, i.e., equal to its own antiparticle (à la π^0). [1] When $\tilde{\nu} = \nu$ the process

$$(A, Z) \longrightarrow (A, Z + 1) + e^- + \tilde{\nu} \equiv (A, Z + 1) + e^- + \nu$$
$$\longrightarrow (A, Z + 2) + 2e^- \tag{3}$$

is also allowed. In absence of the helicity suppression (as would be natural before the observation of parity violation) this neutrinoless ($\beta\beta_{0\nu}$) mode is favoured by phase space over the two-neutrino ($\beta\beta_{2\nu}$) mode by a factor of $10^7 - 10^9$: $T_{0\nu} \sim (10^{13} - 10^{15})$ y while $T_{2\nu} \sim (10^{20} - 10^{24})$ y. Several searches for the $\beta\beta$ decay has been made by the early 1950s with the result that $T \overset{\sim}{>} 10^{17}$. This seemed to point that $\tilde{\nu} \neq \nu$ and prompted the introduction of the lepton number L to distinguish ν from $\tilde{\nu}$: $L = +1$ was attributed to e^- and ν and $L = -1$ to e^+ and $\tilde{\nu}$. The assumption that the additive lepton number is conserved then allows the $\beta\beta_{2\nu}$ decay but prohibits the $\beta\beta_{0\nu}$ one, for which $\Delta L = 2$.

But with the discovery in 1957 that the parity is not conserved for the weak interaction it was realized that the Majorana/Dirac character of the neutrino was

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[1] A Dirac particle can be viewed as a combination of two Majorana particles with equal mass and opposite CP properties and their contribution to the $\beta\beta_{0\nu}$ decay cancel.

still in question. If

$$n \longrightarrow p + e^- + \nu_{RH} \tag{4}$$

$$\nu_{LH} + n \longrightarrow p + e^-$$

then the second process in (3) is forbidden because the right handed neutrino has the wrong helicity to be reabsorbed. Therefore the γ_5- invariance of the weak interaction could account for no $\beta\beta_{0\nu}$ decay, regardless of the Dirac or Majorana nature of the neutrino. Otherwise, this decay can be observed only when the lepton number is not conserved and the neutrino is a massive Majorana particle. [2] This event discouraged experimental searches for a long time, but with the development of modern gauge theories the situation began to change. In fact, there are many reasons for the renaissance of interest in $\beta\beta$-decay over the past decade. The most important one is that, if there is any new physics beyond the standard $SU(2)_L \times U(1)$ gauge model of electroweak interactions, the $\beta\beta_{0\nu}$ decay will play a crucial role in shaping the ultimate theory. Moreover, no solid theoretical principle precludes neutrinos from having mass and the most attractive extensions of the standard model require neutrinos to be massive. The theory is neither capable of predicting the scale of neutrino masses any better than it can fix the masses of quarks and charged leptons.

Since the half-lives can be cast in the simple form

$$T_{2\nu}^{-1} = \mathcal{G}_{2\nu}\mathcal{M}_{2\nu}^2, \qquad\qquad T_{0\nu}^{-1} = \mathcal{G}_{0\nu}\mathcal{M}_{0\nu}^2 <m_\nu>^2,$$

(where $\mathcal{G}'s$ and $\mathcal{M}'s$ are, respectively, the phase space factors and nuclear matrix elements and $<m_\nu>$ is the effective neutrino mass) it is clear then that we shall not understand the $\beta\beta_{0\nu}$ decay unless we understand the $\beta\beta_{2\nu}$ decay. The last is one of the slowest process observed so far in nature and offers a unique opportunity for testing the nuclear physics techniques for half-lives $\overset{\sim}{>} 10^{20}$ y. Thus, the comprehension of the $\beta\beta$ transition mechanism cannot but help advance knowledge of physics in general.

It is worth noting that more than 30 $\beta\beta$ decay experiments are underway or in stages of planning and construction. Until now positive evidence of the $\beta\beta_{2\nu}$ decay

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[2] We assume for simplicity that weak interactions with right-handed currents do not play an essential role in the neutrinoless mode.

mode has been found for ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te, ^{130}Te and ^{238}U. Yet, despite the colossal experimental progress the neutrinoless, lepton violating decay, if it exists, has escaped detection until now. [3]

The $\beta\beta$ decays occur in medium-mass nuclei that are rather far from closed shells, and we all know that shell-model calculations are practical only when the number of valence nucleons is relatively small. Therefore, at the present time, the nuclear structure method most widely used is the quasiparticle random phase approximation (QRPA). Within this model the $\beta\beta$-decay amplitudes are very sensitive to the interaction parameter in the particle-particle (PP) channel, usually denoted by g^{pp}. It is still more interesting that, close to the expected value for g^{pp}, the $\beta\beta$ matrix elements go to zero. But when a physical quantity has a zero (or near zero) a conservative law should, very likely, be at its origin. Thus, resorting to a toy model, I will first discuss the general behavior of the nuclear matrix elements [5, 6]. Later, I will show that they have zeros because of the restoration of the isospin and Wigner $SU(4)$ symmetry [4, 7]. This is not surprising since the Fermi (F) and Gamow-Teller (GT) operators $\tau_\pm$ and $\sigma\tau_\pm$, relevant in the $\beta\beta$ decay, are infinitesimal generators of $SU(2)$ and $SU(4)$, respectively. Finally, we use the concept of restoration of these symmetries to fix the PP interaction strengths and to estimate the $\beta\beta$ matrix elements. It can be argued that, the $SU(4)$ symmetry is badly broken in heavier nuclei and that therefore our recipe is quite arbitrary. I will show, yet, that the residual interaction is capable to overcome most of the $SU(4)$ breaking caused by the spin-orbit splitting.

2. General behavior of $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$ in the QRPA

Independently of the nucleus that decays, of the residual interaction that is used, and of the configuration space that is employed, the $\beta\beta$-moments as a function of g^{pp} always exhibit the following features:

(i) The 2ν moments have first a zero and latter on a pole at which the QRPA collapses.

(ii) The zeros and poles of $\mathcal{M}_{0\nu}$ for the virtual states with spin and parity $J^\pi = 1^+$ are strongly correlated with the zeros and poles of $\mathcal{M}_{2\nu}$.

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[3] The $\beta\beta_{0\nu}$ sensitivities have changed from $\sim 5 \times 10^{15}$ in 1948 to $\sim 5 \times 10^{23}$ in 1987.

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(iii) The total $\beta\beta_{0\nu}$ moments also possess zeros but at significantly larger values of g^{pp}.

The behaviour of the $\beta\beta$ moments for several nuclei are illustrated in Fig. 1. These results have been obtained with a δ force, using standard parametrization [7]. Instead of the parameter g^{pp}, I use here the ratio between the triplet and singlet coupling strengths in the PP channel, i.e., $t = v_t^{pp}/v_s^{pp}$. Calculations with finite range interactions yield similar results [2].

I will resort now to the single mode model (SMM) description [5, 6] of the $\beta\beta$-decays in the $^{48}Ca \rightarrow {}^{48}Ti$ and $^{100}Mo \rightarrow {}^{100}Ru$ systems. This is the simplest version of the QRPA with only one intermediate state for each J^π. It allows to express the moments à la Alaga [8], i.e., as the unperturbed matrix elements $\mathcal{M}_{2\nu}^0$ and $\mathcal{M}_{0\nu}^0(J^+)$ multiplied by the effective charges:

$$\mathcal{M}_{2\nu} = \mathcal{M}_{2\nu}^0 \left(\frac{\omega^0}{\omega_{1^+}}\right)^2 \left(1 + \frac{G(1^+)}{\omega^0}\right), \tag{5}$$

$$\mathcal{M}_{0\nu}(J^+) = \mathcal{M}_{0\nu}^0(J^+)\frac{\omega^0}{\omega_{J^+}} \left(1 + \frac{G(J^+)}{\omega^0}\right), \tag{6}$$

Here $G(J^+) \equiv G(pn, pn; J^+)$ are the PP matrix elements (proportional to t (or to g^{pp})), ω^0 is the unperturbed energy, and ω_{J^+} are the perturbed energies. It will be assumed here that the isospin symmetry is strictly conserved, in which case (as it will be seen latter on) $\mathcal{M}_{2\nu}(0^+) = \mathcal{M}_{0\nu}(0^+) \equiv 0$. When the pairing factors are estimated in the usual manner, one gets

$$\omega = \omega^0\sqrt{1 + F(34 + 9F/\omega^0)/25\omega^0 + 16\,G(1 + F/\omega^0)/25\omega^0}, \tag{7}$$

for the single pair configurations $[0f_{7/2}(n)0f_{7/2}(p)]_{J^+}$ in ^{48}Ca and

$$\omega = \omega^0\sqrt{1 + 4F(45 + F/\omega^0)/225\omega^0 + G(270 + 172F/\omega^0 + 49\,G/\omega^0)/225\omega^0}, \tag{8}$$

for $[0g_{7/2}(n)0g_{9/2}(p)]_{J^+}$ in ^{100}Mo. Therefore, while the numerators in Eqs. (5) and (6) depend only on the PP matrix elements, their denominators depend on the particle-hole (PH) matrix elements $F(J^+) \equiv F(pn, pn; J^+)$, as well. The numbers in the last two equations arise from the pairing factors. As illustrated in Fig. 2, the SMM is a fair first-order approximation for the $\beta\beta_{2\nu}$ decays in ^{48}Ca and ^{100}Mo nuclei.

The role played by the ground state correlations (GSC) in building up Eqs. (5) and (6) can be summarized as follows:

(a) The numerator, i.e., the factor $(1 + G/\omega^0)$, comes from the interference between the forward and backward going contributions. These contribute coherently in the PP channel and totally out of phase in the PH channel.

(b) The G^2 and F^2 terms in the denominator are very strongly quenched by the GSC, while the GF term is enhanced by the same effect. In particular, for ^{48}Ca the term quadratic in G does not contribute at all.

It can be stated therefore that, within the SMM and because of the GSC, the $\mathcal{M}_{2\nu}$ matrix element is mainly a bilinear function of $G(1^+)$. Besides, it passes through zero at $G(1^+) = -\omega^0$ and has a pole when $\omega_{1+} = 0$. Similarly, all $\mathcal{M}_{0\nu}(J^+)$ moments turn out to be quotients of a linear function of $G(J^+)$ and the square root of another linear function of $G(J^+)$. Both the zero and the pole of $\mathcal{M}_{0\nu}(1^+)$ matrix element coincide with those of the 2ν moment. Besides, as the magnitudes of $G(J)$ and $F(J)$ decrease fairly rapidly with J (see Table 1), the quenching effect, induced by the PP interaction, mainly concerns the allowed 0ν moment. For higher order multipoles it could be reasonable to expand the denominator in Eq. (6) in powers of $G(J^+)/\omega^0$ and to keep only the linear term. This term strongly cancels with a similar term in the numerator and the net result is a weak linear dependence of the $\mathcal{M}_{0\nu}(J^+ \neq 1^+)$ moments on the PP strength. Obviously, for the last approximation to be valid, the parameter t (or g^{pp}) has to be small enough to keep ω_{1+} real. Briefly, the SMM can account for all four points raised above, and leads to the following approximations

$$\mathcal{M}_{2\nu} \cong \mathcal{M}_{2\nu}(t = 0)\frac{1 - t/t_0}{1 - t/t_1}, \tag{9}$$

and

$$\begin{aligned}\mathcal{M}_{0\nu} \cong{} & \mathcal{M}_{0\nu}(J^\pi = 1^+; t = 0)\frac{1 - t/t_0}{\sqrt{1 - t/t_1}} \\ & + \mathcal{M}_{0\nu}(J^\pi \neq 1^+; t = 0)(1 - t/t_2), \end{aligned} \tag{10}$$

where $t_1 \geq t_0$ and $t_2 \gg t_1$, and the condition $t \leq t_1$ is fulfilled. It is self evident that these formulae do not depend on the type of residual interaction, and that analogous expressions are obtained when the parameter g^{pp} is used (with g^{pp}'s for t's).

The common behavior of the $\beta\beta$ moments for all nuclei, together with the similarity between the SMM and the full calculations for ^{48}Ca and ^{100}Mo (shown in Figs. 1 and 2, respectively), suggests to go a step further and try to express the exact calculations within the framework of Eqs. (9) and (10). At a first glance this seems a difficult task, because: (i) the SMM does not include the effect of the spin-orbit splitting, which plays a very important role in the $\beta\beta$-decay through the dynamical breaking of the SU(4) symmetry, and (ii) the full calculations involve a rather large configuration space (of the order of 50 basis vectors).

The parameters t_0, t_1, and t_2 that fit the $\beta\beta$ moments displayed in Fig. 1 are listed in Table 2, together with moments $\mathcal{M}_{2\nu}$, $\mathcal{M}_{0\nu}(J^\pi = 1^+)$, and $\mathcal{M}_{0\nu}(J^+ \neq 1^+)$ for $t = 0$. The reliability of formulae (9) and (10) is surprising, to the extent that it is not possible to distinguish visually the exact curves from the fitted ones. It is worth noting that this situation persists even within the number projected QRPA [9]. Why the exact calculations can be accounted for by Eqs. (9) and (10)? I do not know a fully convinced answer. Yet, let me note that for a n dimensional configuration space, $\mathcal{M}_{2\nu}$ can always be expressed by the ratio of the polynomials of degrees $2n - 1$ and $2n$ in $G(1^+)$ [4], i.e.,

$$\mathcal{M}_{2\nu} \cong \mathcal{M}_{2\nu}(t = 0)\frac{1 - t/t_0^{(1)} - t^2/t_0^{(2)} - \cdots - t^{2n-1}/t_0^{(2n-1)}}{1 - t/t_1^{(1)} - t^2/t_1^{(2)} - \cdots - t^{2n}/t_0^{(2n)}}, \tag{11}$$

Thus the above results seem to indicate that cancellations of the type (a) and (b) are likely to be operative to all orders, and that linear terms in $G(1^+)$ are again the dominant ones. General expressions for $\mathcal{M}_{0\nu}$, analogous to (11), are not known, but some cancellation must be taking place in these as well.

3. Restoration of the isospin and SU(4) symmetries

An important question in the QRPA calculations is, how to fix g^{pp} or t? Several attempts have been made to calibrate g^{pp} using the experimental data for individual GT positron decays. The weak point of this procedure is that the distribution of the β^+ strength among low-lying states in odd-odd nuclei is certainly affected by the charge-conserving vibrations, which are not included in the QRPA. For example, the single beta transitions $^{100}Tc \rightarrow {}^{100}Mo$ and $^{100}Tc \rightarrow {}^{100}Ru$ have been discussed

recently in the standard QRPA [10], where the ^{100}Tc states are described as pure pn-quasiparticle excitations, while the suggested wave function for the ground state in ^{101}Mo is (cf. ref. [11]) is only $\approx 35\%$ of quasiparticle nature:

$$| \, 1/2^+\rangle \;\; = \;\; 0.59 \mid s_{1/2}, 00\rangle - 0.57 \mid s_{1/2}, 20\rangle + 0.32 \mid s_{1/2}, 40\rangle$$
$$- \;\; 0.26 \mid d_{5/2}, 22\rangle - 0.26 \mid d_{1/2}, 42\rangle - 0.21 \mid g_{7/2}, 24\rangle$$

(In the basis state $\mid j, NI\rangle$ the quasiparticle j and the N bosons of angular momentum I are coupled to the total spin $1/2$.)

We gauge t by resorting to the restoration of the Wigner SU(4) symmetry [4]. Unlike the method mentioned above, this method involves the total GT strength, which dependent of the charge-conserving vibrations only very weakly. We are aware, however, that the SU(4) symmetry is badly broken in medium and heavy nuclei, and therefore before proceeding, it is necessary to specify what we mean by reconstruction of this symmetry.

For a system with $N \neq Z$, the isospin and spin-isospin symmetries are violated in the mean field approximation, even if the nuclear hamiltonian commutes with the corresponding excitation operators $\beta^{\pm}$ ($\tau_{\mp}$ and $\sigma\tau_{\mp}$). But, we know that when a non-dynamical violation occurs in the BCS-Hartree Fock (BCS-HF) solution, the QRPA induced GSC can be invoked to restore the symmetry. There are subtleties involved in the restoration mechanism: the GSC are not put in evidence explicitly, but only implicitly via their effects on the one-body moments $\beta^{\pm}$ between the ground state and the excited states. Besides, for the F excitations and when the isospin non-conserving forces are absent, a self-consistent inclusion of the GSC leads to the following:

1) all the β^- strength is concentrated in the collective state, and

2) the β^+ spectrum, which in QRPA can be viewed as an extension of the β^- spectrum to negative energies, is totally quenched.

The self-consitency is only attained when the same $S = 0$, $T = 1$ interaction coupling strengths are used in the pairing and PP channels, i.e., when $v_s^{pair} = v_s^{pp}$, and the extent to which the above conditions are fulfilled may be taken as a measure of the isospin symmetry restoration. In Fig. 4 is shown the behavior F strength β^+ as a function of the parameter $s = v_s^{pp}/v_s^{pair}$.

Besides being spontaneously broken by the HF-BCS approximation, the SU(4) symmetry is also dynamically broken by the spin-orbit field and the supermultiplet destroying residual interactions. But, the last two effects have a tendency to cancel each other. In fact, within the TDA the energy differences between the GT and F resonances can be expressed as [12]

$$E_{GT} - E_F = \left[\Delta_{ls} - \left(v_t^{ph} - v_s^{ph} \right) \frac{N - Z}{2A} \right] \, MeV, \tag{12}$$

where $\Delta_{ls} \approx 20A^{-1/3}$ is the mean spin-orbit splitting and v_s^{ph} and v_t^{ph} are, respectively, the singlet and the triplet coupling constants in the PH channel. As $v_t^{ph} > v_s^{ph}$ the residual interaction displaces the GT resonance towards the IAS with increasing $N - Z$. What is more, the energetics of the GT resonances are nicely reproduced by (see Fig. 5)

$$E_{GT} - E_F = \left(26A^{-1/3} - 18.5 \frac{N - Z}{A} \right) \, MeV, \tag{13}$$

which has the same mass and neutron excess dependence as (12). Briefly, the experimental data show that the SU(4) symmetry destroyed by the mean field is partially restored by the residual interaction. [4] The GSC are likely to alter Eq. (12) very little. But, within the QRPA the $\sigma\tau_+$ transition strength is strongly quenched and the GT resonance is somewhat narrowed, as compared with the TDA results. As such the global effect of the pn residual interaction on the GT strengths $\beta^{\pm}$ $(\sigma\tau_{\mp})$ is qualitatively similar to the corresponding effect on the F strengths $\beta^{\pm}$ $(\tau_{\mp})$, in the sense that the conditions 1) and 2) are approximately fulfilled, and we say that the SU(4) symmetry is partially restored. It seems reasonable then to assume that the maximal restoration is achieved for the value of t where the GT strength β^+ is minimum, and this is the way how we fix the parameter t.

4. Results for $\mathcal{M}_{\in\nu}$ and $< m_\nu >$

From the results displayed in Table 3, it can be said that with $t = t_{sym}$ the calculated $\mathcal{M}_{\in\nu}$ moment for ^{48}Ca does not contradict the experimental limit and that the 2ν measurement in ^{82}Se is well accounted for by the theory. On the other

[4]Neither in ^{208}Pb, where the GT strength is located at the energy of the IAS, the SU(4) symmetry is totally restored, as indicated by the resonance width of $\approx 4MeV$.

hand, the calculated 2ν matrix elements turn out to be too small for ^{76}Ge and ^{100}Mo and too large for ^{128}Te and ^{130}Te (in both the cases by a factor of ≈ 3). Yet, one should bear in mind that: i) the calculated values of $\mathcal{M}_{\in\nu}$ vary rather abruptly near $t = t_{sym}$ and therefore it is possible to account for the $\mathcal{M}_{\in\nu}$ in all the cases with a comparatively small variation ($< 10\%$) of t, and ii) the minimum value of the GT β^+ strength critically depends on the spin-orbit splitting over which we still do not have a complete control.

Besides the issue of the procedure adopted for fixing the particle-particle strength parameter within the QRPA, there are some additional problems in calculations of the matrix element $\mathcal{M}_{2\nu}$, as yet not fully understood. They are related with the type of force, choice of the single particle spectra, treatment of the difference between the initial and final nuclei, etc. All these things are to some extent uncertain and therefore it is open to question whether it is possible, at present, to obtain a more reliable theoretical estimate for the 2ν half lives that the one reported here.

The upper limits for the effective neutrino mass $< m_\nu >$, obtained from the measured 0ν half-lives and the calculated matrix elements are shown in Table 4, where also are presented the results obtained by other groups. The difference in a factor of about $2-3$ between both: i) the results obtained by the Pasadena group and the groups of Tübingen and Heidelberg for ^{76}Ge and ^{82}Se nuclei, and ii) the previous and present calculations for ^{100}Mo, ^{128}Te and ^{130}Te nuclei, is just a reflection of the unavoidable uncertainty of the QRPA calculations, and it is difficult to assess which one is "better" and which is "worse".

Acknowledgments

References

[1] See W.C. Haxton and G.J. Stephenson, Prog. Part. Nucl. Phys. **12** (1984) 409; M. Doi, T. Kotani and E. Tagasuki, Prog. Theor. Phys. Suppl. **83** (1985) 1;

F.T. Avignone III and R.L. Brodzinski, in *Neutrinos*, edited by H.V. Klapdor (Springer-Verlag, New-York, 1988), p. 147; K. Muto and H.V. Klapdor, *ibid.* p. 183; F. Krmpotić, in *Lectures on Hadron Physics*, Ed. by E. Ferreira (World Scientific, Singapore, 1990) p. 205

[2] T. Tomoda, Rep. Prog. Phys. **54** (1991) 53, and references therein

[3] J. Kramp, D. Habs, R. Kroth, M. Music, J. Schirmer, D. Schwalm and C. Broude, Nuc. Phys. **A474** (1987) 412

[4] J Hirsch and F. Krmpotić, Phys. Rev. **C41** (1990) 792; J. Hirsch, E. Bauer and F. Krmpotić, Nucl. Phys. **A516** (1990) 304; J. Hirsch and F. Krmpotić, Phys. Lett. **B246** (1990) 5

[5] F. Krmpotić, J. Hirsch and H. Dias, Nucl. Phys. **A542** (1992) 85

[6] F. Krmpotić Phys. Rev. **C48** (1993) 1452

[7] F. Krmpotić and S. Shelly Sharma, Nucl. Phys. **A** (to be published)

[8] G. Alaga, F. Krmpotić, V. Lopac, V. Paar and L. Šips, Fizika **4** (1971) 25

[9] F. Krmpotić, A. Mariano, T.T.S. Kuo and K. Nakayama, Phys. Lett. **B319** (1993) 393

[10] A. Griffiths and P. Vogel, Phys. Rev. **C46** (1992) 181

[11] H. Seyfarth et al. Z. Phys. **A339** (1991) 269

[12] K. Nakayama, A. Pio Galeão and F. Krmpotić, Phys. Lett. **B114** (1982) 217

[13] J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. **C37**, 731 (1988).

[14] K. Muto and H.V. Klapdor, Phys. Lett. **201B**, 420 (1988); K. Muto, E. Bender and H.V. Klapdor-Kleingrothaus, Z. Phys. **A339**, 435 (1991).

[15] J. Suhonen, S.B. Khadkikar and A. Faessler, Nucl. Phys. **A535**, 509 (1991).

Figure Captions

Figure 1: Calculated matrix elements $\mathcal{M}_{2\nu}$ (in units of $[MeV]^{-1}$), the 0ν moments for $J^{\pi} = 1^{+}$ ($\mathcal{M}_{0\nu}(J^{\pi} = 1^{+})$) and total moments $\mathcal{M}_{0\nu}$ as a function of the particle-particle $S = 1$, $T = 0$ coupling constant t.

Figure 2: Exact (solid lines) and SMM (dashed lines) matrix elements $\mathcal{M}_{2\nu}$ (in units of $[MeV]^{-1}$), as a function of the coupling constant t/t_0. t_0 is the value of t for which $\mathcal{M}_{2\nu}$ is null.

Figure 3: Calculated double beta decay matrix elements $\mathcal{M}_{2\nu}$ (in units of $[MeV]^{-1}$) for ^{76}Ge, as a function of t. Solid and dotted curves correspond to the projected (PQRPA) and unprojected (QRPA) results, respectively.

Figure 4: Fermi and Gamow-Teller transition strengths β^{+} for the nuclei ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{128}Te and ^{130}Te, as a function of particle-particle couplings s ($S = 0$, $T = 1$) and t ($S = 1$, $T = 0$) respectively.

Figure 5: Plot of $E_{GT} - E_F$ versus $(N - Z)/A$. When the experimental results overlap (for $^{90,92}Zr$ and ^{208}Pb) we displace them slightly with resect to the correct value of $(N - Z)/A$ for the sake of clarity. The values calculated by Eq. (12) are indicated by full circles.

Tables

Table 1: The $\mathcal{M}^0_{0\nu}(J^+)$ moments and the factors $G(J^+)/\omega^0$ within the single mode model for ^{48}Ca and ^{100}Mo.

J	^{48}Ca $-\mathcal{M}^0_{0\nu}$	$-G(J^+)/\omega^0$	^{100}Mo $-\mathcal{M}^0_{0\nu}$	$-G(J^+)/\omega^0$
0	1.0159	s		
1	1.3439	$\frac{11}{21}t$	2.8316	$\frac{20}{27}t$
2	0.1573	$\frac{5}{21}s$	0.2741	$\frac{20}{77}(t+\frac{2}{27}s)$
3	0.2143	$\frac{19}{77}t$	0.2086	$\frac{130}{693}t$
4	0.0446	$\frac{9}{77}s$	0.1263	$\frac{162}{1001}(t+\frac{20}{81}s)$
5	0.1081	$\frac{235}{1001}t$	0.0585	$\frac{124}{1287}t$
6	0.0122	$\frac{25}{429}s$	0.0842	$\frac{20}{143}(t+\frac{14}{27}s)$
7	0.0988	$\frac{175}{429}t$	0.0177	$\frac{190}{3861}t$
8			0.0925	$\frac{490}{2431}(t+\frac{8}{9}s)$

Table 2: The coefficients t_0, t_1, and t_2 and the matrix elements $\mathcal{M}_{2\nu}$, $\mathcal{M}_{0\nu}(J^\pi = 1^+)$, and $\mathcal{M}_{0\nu}(J^\pi \neq 1^+)$ for $t = 0$, in the parametrization of the 2ν and 0ν $\beta\beta$ moments. The matrix elements $\mathcal{M}_{2\nu}$ are given in units of $[MeV]^{-1}$. The values of the PP coupling strengths, which lead to maximal restoration of the SU(4) symmetry ($t = t_{sym}$), are shown in the last row.

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
$-\mathcal{M}_{2\nu}$	0.173	0.308	0.321	0.451	0.381	0.331
t_0	1.394	1.161	1.206	1.469	1.265	1.261
t_1	1.754	1.680	1.691	1.649	2.131	2.268
$-\mathcal{M}_{0\nu}(J^\pi = 1^+)$	1.506	4.242	4.179	5.015	4.599	4.182
$-\mathcal{M}_{0\nu}(J^\pi \neq 1^+)$	1.501	6.924	7.495	9.762	7.997	7.486
t_0	1.227	1.155	1.141	1.372	1.377	1.407
t_1	1.768	1.741	1.764	1.711	2.236	2.345
t_2	12.82	13.23	12.14	6.527	13.39	11.08
t_{sym}	$\cong 1.50$	$\cong 1.25$	$\cong 1.30$	$\cong 1.50$	$\cong 1.40$	$\cong 1.40$

Table 3: Experimental and calculated 2ν moments for $t = t_{sym}$ (in units of $[MeV]^{-1}$).

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
$\lvert\mathcal{M}_{2\nu}\rvert^{exp}$	< 0.081	$0.280^{+0.006}_{-0.010}$	$0.141^{+0.004}_{-0.014}$	$0.294^{+0.029}_{-0.033}$	$0.038^{+0.01}_{-0.01}$	$0.027^{+0.01}_{-0.01}$
$\mathcal{M}^{cal}_{2\nu}$	0.091	0.100	0.121	0.102	0.118	0.096

Table 4: Upper bounds on the effective neutrino mass $< m_\nu >$ (in eV) obtained from the QRPA calculations of the nuclear matrix elements. For the sake of comparison, in all the cases the same experimental data, as well the same effective axial vector coupling constant ($g_A = -g_V$) have been used.

		^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{128}Te	^{130}Te
Pasadena	(ref. [13])		4.4	20	20	1.8	22
Heidelberg	(ref. [14])	22	2.0	7.4	26	1.5	21
Tübingen	(ref. [15])		3.1	12		3.8	31
our results	(ref. [7])	71	1.5	5.3	8.8	1.0	12

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